# Bulgarian Mathematical Olympiad 1966, III Round

*First Day*

1. Find all possible values of the natural number $n$ for which the number $n^{n+1} - (n+1)^n$ is divisible by 3. (6 Points)

2. Prove the inequality:

$$\log_{b+c} a^2 + \log_{c+a} b^2 + \log_{a+b} c^2 \geq 3$$

where the numbers $a$, $b$, $c$ are not smaller than 2. (8 points)

3. In the plane are given $n$ points. It is known that if we choose any four of these points there are three points that lie on a common straight line. Prove that all n points maybe except one lie on a common straight line.

(6 points)

*Second day*

4. It is given a tetrahedron $ABCD$. Medians of the triangle $BCD$ meets each other in point $M$. Prove the inequality:

$$AM \leq \frac{AB+AC+AD}{3}$$

(5 Points)

5. In the triangle $ABC$ the angle bisector, median and the height drawn respectively through the vertices $A$, $B$, $C$ intersects at a common point. Prove that the angles $A$, $B$, $C$ satisfies the equation:

$$\tan A = \frac{\sin B}{\cos C}$$

(9 Points)

6. In a tourist tour participates yang people, girls and boys. It is known that every boy knows at least one girl but he doesn't know all the girls, and every girl knows at least one of the boys but she doesn't know all the boys. Prove that from participants may be chosen two boys and two girls such that each of the selected boys knows one of the selected girls but doesn't know the other selected girl and each of the selected girls knows one of the selected boys but doesn't know the other selected boy. (6 Points)



The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
LaTeX by Borislav Mirchev
www.imomath.com