Bulgarian Mathematical Olympiad 1960, III Round

First Day

1. Prove that the sum (and/or difference) of two irreducible fractions with different divisors cannot be an integer number. (7 points)

2. Find minimum and the maximum of the function:

$$y = \frac{x^2 + x + 1}{x^{+} 2x + 1}$$

if x can achieve all possible real values. (6 points)

3. Find tan of the angles: x, y, z from the equations: $\tan x : \tan y : \tan z = a : b : c$ if it is known that $x + y + z = 180^\circ$ and a, b, c are positive numbers. (7 points)

Second day

4. There are given two externally tangent circles with radii R and r.

   (a) prove that the quadrilateral with sides - two external tangents to and chords, connecting tangents of these tangents is a trapezium;

   (b) Find the bases and the height of the trapezium.

   (6 points)

5. The rays a, b, c have common starting point and doesn't lie in the same plane. The angles $\alpha = \angle(b,c)$, $\beta = \angle(c,a)$, $\gamma = \angle(a,b)$, are acute and their dimensions are given in the drawing plane. Construct with a ruler and a compass the angle between the ray a and the plane, passing through the rays b and c. (8 points)

6. In a cone is inscribed a sphere. Then it is inscribed another sphere tangent to the first sphere and tangent to the cone (not tangent to the base). Then it is inscribed third sphere tangent to the second sphere and tangent to the cone (not tangent to the base). Find the sum of the surfaces of all inscribed spheres if the cone's height is equal to h and the angle throught a vertex of the cone formed by a intersection passing from the height is equal to α. (6 points)

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