1. Let $ABCD$ be a convex quadrilateral. Prove that the incircles of triangles $ABC$, $BCD$, $CDA$ and $DAB$ have a common point if and only if $ABCD$ is a rhombus.

2. Determine all positive integers $n$ for which it is possible to divide a triangle in $n$ non-degenerate triangles in such a way that each of the vertices of the triangles is an endpoint of the same number of sides of the triangles.

3. Let $x_1, x_2, \ldots, x_{2004}$ be a sequence of integers satisfying

\[ x_{k+3} = x_{k+2} + x_{k+1} + x_k \quad \text{for each} \quad k = 1, 2, \ldots, 2001. \]

Can more than a half of the terms of this sequence be negative?

4. Consider all $10 \times 10$ tables with entries $0, 1, \ldots, 9$ which contain each of the numbers $0, \ldots, 9$ exactly $10$ times. Find the greatest $n$ such that, in every such table, there is a row or column containing at least $n$ distinct entries.

5. Consider the sequence $(a_n)$ given by $a_0 = a_1 = a_2 = a_3 = 1$ and

\[ a_n a_{n-4} = a_{n-1} a_{n-3} + a_{n-2}^2. \]

Prove that all its terms are integers.

6. Let $a, b$ be real numbers. A function $f_{a,b} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by

\[ f_{a,b}(x,y) = (a - by - x^2, x). \]

For each $P \in \mathbb{R}^2$, define $f_{a,b}^0(P) = P$ and $f_{a,b}^{k+1}(P) = f_{a,b}(f_{a,b}^k(P))$, where $k$ is a nonnegative integer. The set $\text{per}(a,b)$ of periodic points of $f_{a,b}$ is defined as the set of all points $P \in \mathbb{R}^2$ for which there exists $n \in \mathbb{N}$ such that $f_{a,b}^n(P) = P$.

For a fixed real $b$, show that the set $A_b = \{ a \in \mathbb{R} \mid \text{per}(a,b) \neq \emptyset \}$ has the smallest element. Determine this smallest element.