# 13-th Bosnia and Hercegovina Mathematical Olympiad 2008

Sarajevo, May 17–18, 2008

*First Day*

1. Given an isosceles triangle $ABC$ with $AC = BC = b$, prove that $b > \pi r$, where $r$ is the inradius.

2. Find all pairs $(m,n)$ of positive integer that satisfy the following two conditions:

   (i) $m^2 - n \mid m + n^2$;

   (ii) $n^2 - m \mid n + m^2$.

3. 30 persons are sitting at a round table. $30 - N$ of them always tell truth (let us call them *truth tellers*) while the other $N$ sometimes tell truth and sometimes lie (we will call them *liars*). The question "Who is your right neighbor: truth-teller or a liar?" is asked to all 30 persons. What is the maximal $N$ for which the obtained answers will always guarantee that we are able to find at least one truth teller?

*Second Day*

4. Eight students worked on eight problems. It turned out that each problem was solved by at least 5 students. Prove that it is always possible to find two students so that each problem was solved by at least one of them.

5. Let $AD$ be the altitude from $A$ of $\triangle ABC$ and let $R$ be the circumradius. Let $E$ and $F$ be the feet of perpendiculars from $D$ to $AB$ and $AC$. If $AD = R\sqrt{2}$, prove that $EF$ passes trough the circumcenter of $\triangle ABC$.

6. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(f(x)+y) = f(x^2 - y) + 4f(x)y$$

for all $x, y \in \mathbb{R}$.



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