8-th Bosnia and Herzegovina Mathematical Olympiad

Neum, May 10–11, 2003

First Day

1. Initially, the numbers 5, 7 and 9 are written on a blackboard. In each step, we choose numbers a, b from the blackboard with $a > b$ and write the number $5a - 4b$ on the blackboard. Is it possible to obtain number 2003 in a number of such operations?

2. On the sides AB and BC of a triangle ABC are externally constructed squares ABB_1A_1 and BCC_1B_2. Prove that the lines AC_1 and CA_1 meet at a point on the altitude from B.

3. For every natural number n prove the inequality

$$(n-1)^n + 2n^n \leq (n+1)^n \leq 2(n-1)^n + 2n^n.$$

Second Day

4. In a triangle ABC, AD and BE are altitudes and L the feet of the perpendicular from B to DE. Prove that if $LB^2 = LD \cdot LE$ then triangle ABC is isosceles.

5. Given a regular $2n$-gon with center S, consider all quadrilaterals with the vertices at vertices of the $2n$-gon. Denote by u the number of such quadrilaterals containing S in its interior and by v the number of remaining quadrilaterals. Determine $u - v$.

6. Real numbers a, b, c satisfy $|a| \geq 2$ and $a^2 + b^2 + c^2 = abc + 4$. Show that there exist real numbers x and y such that

$$a = x + \frac{1}{x}, \quad b = y + \frac{1}{y}, \quad c = xy + \frac{1}{xy}.$$

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