6-th Bosnia and Hercegovina Mathematical Olympiad 2001

Lukavica, May 19–20, 2001

First Day

1. Points A, B, C on a given circle divide the circle into the arcs whose lengths are in ratio $3:5:7$. Compute the angles of $\triangle ABC$.

2. Positive integers satisfy the equality $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$. Show that $xyz \geq 3600$.

3. Determine the maximum natural number n for which there exists an n-element subset S of set $\{1,2,\dots,2001\}$ such that the equation $y=2x$ has no solution in S.

Second Day

4. Two circles with radii r_1 and r_2, exterior to each other, are given in the plane. Their interior common tangent intersects the two exterior common tangents at points A and B and touches one of the circles at C. Prove that $AC \cdot BC = r_1 r_2$.

5. Let $x_1, x_2, \dots, x_n$ be $n>1$ positive numbers with the sum 1. Decide whether the following inequality is necessarily true:

$$\sum_{i=1}^{n} \frac{x_i}{1-x_1x_2\cdots x_{i-1}x_{i+1}\cdots x_n} \leq \frac{1}{1-\left(\frac{1}{n}\right)^{n-1}}.$$

6. Prove that there exist infinitely many positive integers n for which the equation

$$(x+y+z)^3 = n^2xyz$$

has a solution (x,y,z) in positive integers.

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