Flanders Mathematical Olympiad 1997

Final Round

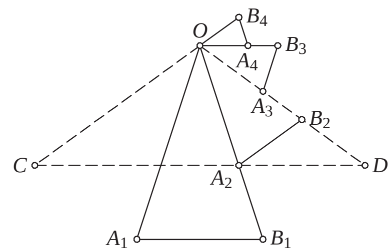
1. Write 1997 as a sum of natural numbers whose product is as large as possible.

2. Consider the curves given by $x^2 + y^2 = r^2$ (with $r > 0$ given) and $(xy)^2 = 1$. Denote by F_r the convex polygon with the vertices at the intersection points (asuming that these exist) of these two curves.

   (a) Compute the area $f(r)$ of the polygon F_r.

   (b) For which values of r is F_r a regular polygon?

3. An isosceles triangle $\triangle_1 = OA_1B_1$ with the top angle $\angle A_1OB_1 = 36^\circ$ is given. We construct equally oriented triangles $\triangle_n = OA_nB_n$ as follows: for $n > 1$, A_n lies on segment OB_{n-1}, $OA_n = OB_n = A_{n-1}B_{n-1}$, and $\angle A_nOB_n = 36^\circ$. Prove that the area of the union of all triangles constructed in this way does not exceed the area of the triangle OCD with $\angle COD = 108^\circ$ and $OC = OD = OA_1$.



4. Thirteen birds are siting on a flat surface in such a way that among any five of them, four are sitting on a circle.

   (a) Prove that there are six birds sitting on a circle.

   (b) Can this result be sharpened?

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