Flanders Mathematical Olympiad 1996

Final Round

1. Let ABC and DAC be two isosceles triangles with the top angles $\angle BAC = 20^\circ$ and $\angle ADC = 100^\circ$. Show that $AB = BC + CD$.

2. Let P be the set of prime numbers greater than 5. Find the greatest common divisor of the numbers $p^8 - 1$, where $p \in P$.

3. Consider the points $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$ on the real line. Find the smallest length l such that all these points can be covered with five segments of length l.

4. Consider a real polynomial $p(x) = a_n x^n + \dots + a_1 x + a_0$.

   (a) Prove that if $\deg p(x) \geq 2$, then $\deg p(x) = 2 + \deg(p(x+1) + p(x-1) - 2p(x))$.

   (b) Suppose $p(x)$ is polynomial for which there are real constants r and s such that $p(x+1) + p(x-1) - rp(x) - s = 0$ for all x. Prove that there are $a, b, c \in \mathbb{R}$ such that $p(x) = a + bx + cx^2$.

   (c) Prove that in (b), $s = 0$ implies $c = 0$.

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