Flanders Mathematical Olympiad 1993

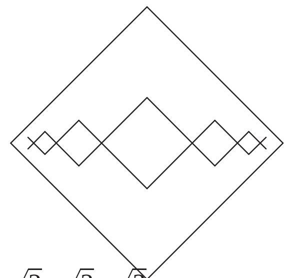
Final Round

1. In December, each of the 20 students in a class sends 10 greeting cards to 10 different classmates (of course, not to himself).

   (a) Show that at least two students sent each other a greeting card.

   (b) Now suppose there are n students in a class, each of them sending greeting cards to m different classmates. For which m and n do there necessarily exist two students sending each other a greeting card?

2. A jeweler covers the diagonal of a unit square with small golden squares in the following way: the sides of all squares are parallel to the sides of the unit square, each square has a side equal to either half or double of the size of it's neighbouring square (squares are neighbours if they share a vertex), each center of a square has distance to the vertex of the unit square equal to $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{8}, \ldots$, and all centers are on the diagonal of the initial square (see the picture).

   (a) What is the side length of the middle square?

   (b) What is the total gold-plated area?

3. If a, b, c are positive numbers, prove the inequality

$$-1 < \left(\frac{a-b}{a+b}\right)^{1993} + \left(\frac{b-c}{b+c}\right)^{1993} + \left(\frac{c-a}{c+a}\right)^{1993} < 1.$$

4. Let b a line perpendicular to line a_0 at point O, and for $n \geq 0$, let a_{n+1} be the bisector of the acute angle between the lines a_n and b. Point A_0 with $OA_0 = 1$ is taken on a_0, and for all $n \geq 0$, A_{n+1} is the orthogonal projection of A_n onto a_{n+1}. Determine $\lim_{n \to +\infty} OA_n$.

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