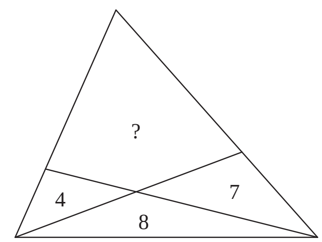
Flanders Mathematical Olympiad 2001

Final Round

1. Prove that for every natural number $n > 1$, $(n-1)^2$ divides $n^{n-1} - 1$.

2. Two straight lines through vertices divide a triangle into four pieces. The areas of of three of the pieces are shown on the picture. Determine the area of the fourth piece (denoted by ”?”).



3. A regular 2001-gon is inscribed in a circle. Consider a regular 667-gon whose vertices are at vertices of the 2001-gon. Prove that the area of the part of the 2001-gon lying outside the 667-gon equals

$$k \sin^3 \frac{\pi}{2001} \cos^3 \frac{\pi}{2001}$$

for some positive integer k, and determine k.

4. A student is solving quadratic equation as follows. He starts with a first quadratic equation $x^2 + ax + b = 0$ with a and b nonzero and finds its solutions p, q. If p and q are real with $p \leq q$, he forms the second quadratic equation $x^2 + px + q = 0$. He continues this process as long as possible. Prove that he will stop at latest at the fifth equation.

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