

24-th Balkan Mathematical Olympiad

Rhodes, Greece – April 28, 2007

1. In a convex quadrilateral $ABCD$ with $AB = BC = CD$, the diagonals AC and BD are of different length and intersect at point E . Prove that $AE = DE$ if and only if $\angle BAD + \angle ADC = 120^\circ$. *(Albania)*

2. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that for all real x, y ,

$$f(f(x) + y) = f(f(x) - y) + 4f(x)y. \quad (\text{Bulgaria})$$

3. Determine all natural numbers n for which there exists a permutation σ of numbers $1, 2, \dots, n$ such that the number

$$\sqrt{\sigma(1) + \sqrt{\sigma(2) + \sqrt{\dots + \sqrt{\sigma(n)}}}}$$

is rational. *(Serbia)*

4. Let $n \geq 3$ be an integer. Let $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ be the circumferences of three convex n -gons in a plane such that the intersection of any two of them is a finite set of points. Find the maximum possible number of points in $\mathcal{C}_1 \cap \mathcal{C}_2 \cap \mathcal{C}_3$. *(Turkey)*