22-nd Balkan Mathematical Olympiad

Iaşi, Romania – May 6, 2005

1. The incircle of an acute-angled triangle ABC touches AB at D and AC at E. Let the bisectors of the angles $\angle ACB$ and $\angle ABC$ intersect the line DE at X and Y respectively, and let Z be the midpoint of BC. Prove that the triangle XYZ is equilateral if and only if $\angle A = 60^\circ$. *(Bulgaria)*

2. Find all primes p such that $p^2 - p + 1$ is a perfect cube. *(Albania)*

3. If a, b, c are positive real numbers, prove the inequality

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq a + b + c + \frac{4(a-b)^2}{a+b+c}.$$

   When does equality occur? *(Serbia and Montenegro)*

4. Let $n \geq 2$ be an integer, and let S be a subset of $\{1, 2, \dots, n\}$ such that S neither contains two coprime elements, nor does it contain two elements, one of which divides the other. What is the maximum possible number of elements of S? *(Romania)*

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