17-th Balkan Mathematical Olympiad

Chişinău, Moldova – May 5, 2000

1. [*BMO 1997#4*] Determine all functions $f:\mathbb{R}\to\mathbb{R}$ that satisfy
$$f(xf(x)+f(y)) = f(x)^2+y \quad \text{for all } x,y.$$
(Albania)

2. Let ABC be a scalene triangle and E be a point on the median AD. Point F is the orthogonal projection of E onto BC. Let M be a point on the segment EF, and N,P be the orthogonal projections of M onto AC and AB respectively. Prove that the bisectors of the angles PMN and PEN are parallel.

3. Find the maximal number of rectangles $1\times 10\sqrt{2}$ that can be cut off from a rectangle 50×90 by using cuts parallel to the edges of the big rectangle.
(Yugoslavia)

4. A positive integer is a *power* if it is of the form t^s for some integers $t,s\geq 2$. Prove that for any natural number n there exists a set A of positive integers with the following properties:
   (i) A has n elements;
   (ii) Every element of A is a power;
   (iii) For any $2\leq k\leq n$ and any $r_1,\dots,r_k\in A$, $\dfrac{r_1+\cdots+r_k}{k}$ is a power.

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