15-th Balkan Mathematical Olympiad

Nicosia, Cyprus – May 5, 1998

1. Consider the finite sequence $\left[\frac{k^2}{1998}\right]$, $k = 1, 2, \ldots, 1997$. How many distinct terms are there in this sequence? *(Greece)*

2. Let $n \geq 2$ be an integer, and let $0 < a_1 < a_2 < \cdots < a_{2n+1}$ be real numbers. Prove the inequality
$$\sqrt[n]{a_1} - \sqrt[n]{a_2} + \sqrt[n]{a_3} - \cdots + \sqrt{a_{2n+1}} < \sqrt[n]{a_1 - a_2 + a_3 - \cdots + a_{2n+1}}.$$
(Romania)

3. Let $\mathscr{S}$ denote the set of points inside or on the border of a triangle ABC, without a fixed point T inside the triangle. Show that $\mathscr{S}$ can be partitioned into disjoint closed segemnts. *(Yugoslavia)*

4. Prove that the equation $y^2 = x^5 - 4$ has no integer solutions. *(Bulgaria)*

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