12-th Balkan Mathematical Olympiad

Plovdiv, Bulgaria – May 9, 1995

1. Define $x * y = \dfrac{x+y}{1+xy}$. Evaluate $(\ldots(((2*3)*4)*5)*\ldots)*1995$.

   (FYR Macedonia)

2. Circles $c_1(O_1, r_1)$ and $c_2(O_2, r_2)$, $r_2 > r_1$, intersect at A and B so that $\angle O_1AO_2 = 90^\circ$. The line O_1O_2 meets c_1 at C and D, and c_2 at E and F (in the order $C-E-D-F$). The line BE meets c_1 at K and AC at M, and the line BD meets c_2 at L and AF at N. Prove that
$$\frac{r_2}{r_1} = \frac{KE}{KM} \cdot \frac{LN}{LD}.$$
   (Greece)

3. Let a and b be natural numbers with $a > b$ and $2 \mid a+b$. Prove that the solutions of the equation
$$x^2 - (a^2 - a + 1)(x - b^2 - 1) - (b^2+1)^2 = 0$$
   are natural numbers, none of which is a perfect square. *(Albania)*

4. Let n be a natural number and S be the set of points (x, y) with $x, y \in \{1, 2, \ldots, n\}$. Let T be the set of all squares with the verticesw in the set S. We denote by a_k ($k \geq 0$) the number of (unordered) pairs of points for which there are exactly k squares in T having these two points as vertices. Show that $a_0 = a_2 + 2a_3$. *(Yugoslavia)*

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