11-th Balkan Mathematical Olympiad

Novi Sad, Yugoslavia – May 10, 1994

1. An acute angle XAY and a point P inside it are given. Construct (by a ruler and a compass) a line that passes through P and intersects the rays AX and AY at B and C such that the area of the triangle ABC equals AP^2. *(Cyprus)*

2. Let m be an integer. Prove that the polynomial
$$x^4 - 1994x^3 + (1993 + m)x^2 - 11x + m$$
has at most one integer zero. *(Greece)*

3. Let $(a_1, a_2, \ldots, a_n)$ be a permutation of the numbers $1, 2, \ldots, n$, where $n \geq 2$. Determine the largest possible value of
$$\sum_{k=1}^{n-1} |a_{k+1} - a_k|.$$
(Romania)

4. Find the smallest number $n > 4$ for which there can exist a set of n people, such that any two people who are acquainted have no common acquaintances, and any two people who are not acquainted have exactly two common acquaintances. (Acquaintance is a symmetric relation.)

(Bulgaria)

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com