10-th Balkan Mathematical Olympiad

Nicosia, Cyprus – May 3-8, 1993

1. Let a, b, c, d, e, f be real numbers which satisfy

$$a + b + c + d + e + f = 10,$$
$$(a-1)^2 + (b-1)^2 + (c-1)^2 + (d-1)^2 + (e-1)^2 + (f-1)^2 = 6.$$

   Find the maximum possible value of f. *(Cyprus)*

2. A natural number with the decimal representation $\overline{a_N a_{N-1} \dots a_1 a_0}$ is called *monotone* if $a_N \le a_{N-1} \le \cdots \le a_0$. Determine the number of all monotone numbers with at most 1993 digits. *(Bulgaria)*

3. Circles C_1 and C_2 with centers O_1 and O_2, respectively, are externally tangent at point Γ. A circle C with center O touches C_1 at A and C_2 at B so that the centers O_1, O_2 lie inside C. The common tangent to C_1 and C_2 at Γ intersects the circle C at K and L. If D is the midpoint of the segment KL, show that $\angle O_1OO_2 = \angle ADB$. *(Greece)*

4. Let p be a prime and $m \ge 2$ be an integer. Prove that the equation

$$\frac{x^p + y^p}{2} = \left(\frac{x+y}{2}\right)^m$$

   has a positive integer solution $(x, y) \ne (1, 1)$ if and only if $m = p$.

   (Romania)

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