9-th Balkan Mathematical Olympiad

Athens, Greece – May 4-9, 1992

1. For positive integers m, n, define $A(m,n) = m^{3^{4n}+6} - m^{3^{4n}+4} - m^5 + m^3$. Find all numbers n with the property that $A(m,n)$ is divisible by 1992 for every m. *(Bulgaria)*

2. Prove that for all positive integers n,
$$(2n^2+3n+1)^n \geq 6^n (n!)^2.$$
(Cyprus)

3. Let D, E, F be points on the sides BC, CA, AB respectively of a triangle ABC (distinct from the vertices). If the quadrilateral $AFDE$ is cyclic, prove that
$$\frac{4S_{DEF}}{S_{ABC}} \leq \left(\frac{EF}{AD}\right)^2.$$
(Greece)

4. For each integer $n \geq 3$, find the smallest natural number $f(n)$ having the following property: For every subset $A \subset \{1, 2, \ldots, n\}$ with $f(n)$ elements, there exist elements $x, y, z \in A$ that are pairwise coprime. *(Romania)*

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