8-th Balkan Mathematical Olympiad

Constanţa, Romania – May 13-18, 1991

1. Let M be a point on the arc AB not containing C of the circumcircle of an acute-angled triangle ABC, and let O be the circumcenter. The perpendicular from M to OA intersects AB at K and AC at L. The perpendicular from M to OB intersects AB at N and BC at P. If $KL = MN$, express $\angle MLP$ in terms of the angles of $\triangle ABC$. *(Greece)*

2. Prove that there are infinitely many pairwise non-congruent triangles T such that:

   (i) The sides a, b, c of T are coprime positive integers;

   (ii) The area of T is an integer;

   (iii) None of the altitudes of T is an integer. *(Yugoslavia)*

3. A regular hexagon of area H is inscribed in a convex polygon of area P. Prove that $P \leq \frac{3}{2}H$. When does equality occur? *(Bulgaria)*

4. Prove that there is no bijection $f : \{1,2,\dots\} \to \{0,1,2,\dots\}$ such that

$$f(mn) = f(m) + f(n) + 3f(m)f(n) \quad \text{for all } m,n.$$

(Romania)

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