7-th Balkan Mathematical Olympiad

Sofia, Bulgaria – May 6-11, 1990

1. The sequence (a_n) is given by $a_1 = 1$, $a_2 = 3$, and
$$a_{n+2} = (n+3)a_{n+1} - (n+2)a_n \quad \text{for all } n.$$
Find all terms of the sequence that are divisible by 11. *(Greece)*

2. If $a_0 + a_1x + \cdots + a_{2n}x^{2n} = (x + 2x^2 + \cdots + nx^n)^2$, prove that
$$a_{n+1} + a_{n+2} + \cdots + a_{2n} = \frac{n(n+1)(5n^2+5n+2)}{24}.$$
(Bulgaria)

3. The feet of the altitudes of a non-equilateral triangle ABC are A_1, B_1, C_1. If A_2, B_2, C_2 are the tangency points of the incircle of the triangle $A_1B_1C_1$ with its sides, prove that the Euler lines of the triangles ABC and $A_2B_2C_2$ coincide. *(Yugoslavia)*

4. Determine the smallest number of elements of a finite set A for which there is a function $f : \mathbb{N} \to A$ such that $f(i) \neq f(j)$ whenever $|i - j|$ is a prime number. *(Romania)*

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