5-th Balkan Mathematical Olympiad

Nicosia, Cyprus – May 1-7, 1988

1. Let CH, CL, CM be the altitude, angle bisector, and median of a triangle ABC, respectively, where H, L, M are on AB. Given that the ratios of the areas of $\triangle HMC$ and $\triangle LMC$ to the area of $\triangle ABC$ are equal to $\frac{1}{4}$ and $1-\frac{\sqrt{3}}{2}$, respectively, determine the angles of $\triangle ABC$. *(Bulgaria)*

2. Find all polynomials $P(x,y)$ in two variables such that for all real a,b,c,d,
$$P(a,b)P(c,d) = P(ac+bd, ad+bc).$$ *(Yugoslavia)*

3. Show that every tetrahedron $A_1A_2A_3A_4$ can be placed between two parallel planes which are at the distance at most $\frac{1}{2}\sqrt{\frac{P}{3}}$, where
$$P = A_1A_2^2 + A_1A_3^2 + A_1A_4^2 + A_2A_3^2 + A_2A_4^2 + A_3A_4^2.$$ *(Greece)*

4. Find all pairs (a_n, a_{n+1}) of consecutive terms of the sequence $a_n = 2^n + 49$ such that $a_n = pq$, $a_{n+1} = rs$, where p,q,r,s are prime numbers with $p<q$, $r<s$, and $q-p = s-r$. *(Romania)*

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