4-th Balkan Mathematical Olympiad

Athens, Greece – May 3-8, 1987

1. Let a be a real number. Assume $f:\mathbb{R}\to\mathbb{R}$ is a function such that $f(0)=1/2$ and
$$f(x+y)=f(x)f(a-y)+f(y)f(a-x) \quad \text{for all } x,y\in\mathbb{R}.$$
Prove that f is constant. *(Yugoslavia)*

2. Suppose that $x\geq 1$ and $y\geq 1$ are real numbers such that the numbers $a=\sqrt{x-1}+\sqrt{y-1}$ and $b=\sqrt{x+1}+\sqrt{y+1}$ are non-consecutive integers. Show that $b=a+2$ and $x=y=\frac{5}{4}$. *(Romania)*

3. In a triangle ABC, the angles α,β (at A and B) satisfy
$$\sin^{23}\frac{\alpha}{2}\cos^{48}\frac{\beta}{2}=\sin^{23}\frac{\beta}{2}\cos^{48}\frac{\alpha}{2}.$$
Compute AC/BC. *(Cyprus)*

4. Circles $k_1(O_1,1)$ and $k_2(O_2,\sqrt{2})$ with $O_1O_2=2$ intersect at A and B. Find the length of the chord AC of circle k_2 whose midpoint lies on k_1.
(Bulgaria)

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com