2-nd Balkan Mathematical Olympiad

Sofia, Bulgaria – May 4-9, 1985

1. Let O be the circumcircle of a triangle ABC, D be the midpoint of AB, and E be the centroid of triangle ACD. Prove that $CD \perp OE$ if and only if $AB = AC$. *(Bulgaria)*

2. Assume that $a,b,c,d \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ satisfy

$$\sin a + \sin b + \sin c + \sin d = 1,$$
$$\cos 2a + \cos 2b + \cos 2c + \cos 2d \geq \frac{10}{3}.$$

   Prove that $a,b,c,d \in [0, \frac{\pi}{6}]$. *(Romania)*

3. All integer points of the form $19a + 85b$ ($a,b \in \mathbb{N}$) are colored red, while all other integer points are colored green. Find whether there exists a point A on the real axis, such that any two integer points B,C that are symmetric with respect to A received different colors. *(Greece)*

4. On a conference 1985 poeple take part. In every group of three people, at least two speak the same language. If every person can speak at most five languages, prove that there exist 200 people on the conference which all speak the same language. *(Romania)*

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The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
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