17-th Austrian Mathematical Olympiad 1986

Final Round

First Day

1. Show that a square can be inscribed in any regular polygon.

2. For $s,t \in \mathbb{N}$, consider the set $M = \{(x,y) \in \mathbb{N}^2 \mid 1 \le x \le s,\ 1 \le y \le t\}$. Find the number of rhombi with the vertices in M and the diagonals parallel to the coordinate axes.

3. Find all possible values of x_0 and x_1 such that the sequence defined by

$$x_{n+1} = \frac{x_{n-1}x_n}{3x_{n-1} - 2x_n} \quad \text{for } n \ge 1$$

contains infinitely many natural numbers.

Second Day

4. Find the largest n for which there is a natural number N with n decimal digits which are all different such that $n!$ divides N. Furthermore, for this largest n find all possible numbers N.

5. Show that for every convex n-gon $(n \ge 4)$, the arithmetic mean of the lengths of its sides is less than the arithmetic mean of the lengths of all its diagonals.

6. Given a positive integer n, find all functions $F : \mathbb{N} \to \mathbb{R}$ such that $F(x+y) = F(xy-n)$ whenever $x,y \in \mathbb{N}$ satisfy $xy > n$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com