

## 11-th Asian–Pacific Mathematical Olympiad 1999

1. Find the smallest positive integer  $n$  with the following property: there does not exist an arithmetic progression of 1999 real numbers containing exactly  $n$  integers.
2. Let  $a_1, a_2, \dots$  be a sequence of real numbers satisfying  $a_{i+j} \leq a_i + a_j$  for all  $i, j = 1, 2, \dots$ . Prove that

$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} + \dots + \frac{a_n}{n} \geq a_n \quad \text{for each } n \in \mathbb{N}.$$

3. Let  $\Gamma_1$  and  $\Gamma_2$  be two circles intersecting at  $P$  and  $Q$ . The common tangent, closer to  $P$ , of  $\Gamma_1$  and  $\Gamma_2$  touches  $\Gamma_1$  at  $A$  and  $\Gamma_2$  at  $B$ . The tangent of  $\Gamma_1$  at  $P$  meets  $\Gamma_2$  at  $C \neq P$ , and the extension of  $AP$  meets  $BC$  at  $R$ . Prove that the circumcircle of triangle  $PQR$  is tangent to  $BP$  and  $BR$ .
4. Determine all pairs  $(a, b)$  of integers with the property that the numbers  $a^2 + 4b$  and  $b^2 + 4a$  are both perfect squares.
5. Let  $S$  be a set of  $2n + 1$  points in the plane such that no three are collinear and no four concyclic. A circle will be called *good* if it has three points of  $S$  on its circumference,  $n - 1$  points in its interior and  $n - 1$  points in its exterior. Prove that the number of good circles has the same parity as  $n$ .