7-th Asian–Pacific Mathematical Olympiad 1995

1. Determine all sequences of real numbers $a_1, a_2, \ldots, a_{1995}$ which satisfy:

$$2\sqrt{a_n - (n-1)} \geq a_{n+1} - (n-1) \quad \text{for } n = 1, 2, \ldots 1994, \text{ and}$$
$$2\sqrt{a_{1995} - 1994} \geq a_1 + 1.$$

2. Let $a_1, a_2, \ldots, a_n$ be a sequence of integers with values between 2 and 1995 such that:

   (i) Any two of the a_i's are coprime;

   (ii) Each a_i is either a prime or a product of distinct primes.

   Determine the smallest possible n to make sure that the sequence will contain a prime number.

3. Let $PQRS$ be a cyclic quadrilateral such that the segments PQ and RS are not parallel. Consider the set of circles through P and Q, and the set of circles through R and S. Determine the set $\mathscr{A}$ of points of tangency of circles in these two sets.

4. Let $\mathscr{C}$ be a circle with radius R and center O, and S a fixed point in the interior of $\mathscr{C}$. Let AA' and BB' be perpendicular chords through S. Consider the rectangles $SAMB$, $SBN'A'$, $SA'M'B'$, and $SB'NA$. Find the set of all points M, N', M', and N when A moves around the whole circle.

5. Find the minimum positive integer k such that there exists a function f from the set $\mathbb{Z}$ to $\{1, 2, \ldots, k\}$ with the property that $f(x) \neq f(y)$ whenever $|x - y| \in \{5, 7, 12\}$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com