# 5-th Asian–Pacific Mathematical Olympiad 1993

1. Let $ABCD$ be a rhombus with $\angle ABC = 60^\circ$. Let $l$ be a line passing through $D$ and having no other points in common with the rhombus. Let $E$ and $F$ be the points of intersection of $l$ with $AB$ and $BC$ respectively. Let $M$ be the point of intersection of $CE$ and $AF$. Prove that $CA^2 = CM \cdot CE$.

2. Find the total number of different integer values the function
$$f(x) = [x] + [2x] + [5x/3] + [3x] + [4x]$$
takes for real numbers $x$ with $0 \le x \le 100$.

3. Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$ and $g(x) = c_{n+1}x^{n+1} + \cdots + c_1 x + c_0$ be nonzero real polynomials such that $g(x) = (x+r)f(x)$ for some real number $r$. If $a = \max(|a_n|, \dots, |a_0|)$ and $c = \max(|c_{n+1}|, \dots, |c_0|)$, prove that $\dfrac{a}{c} \le n+1$.

4. Determine all positive integers $n$ for which the equation
$$x^n + (2+x)^n + (2-x)^n = 0$$
has a solution in the integers.

5. Let $P_1, P_2, \dots, P_{1993} = P_0$ be distinct points in the $xy$-plane with the following properties:
   (i) both coordinates of $P_i$ are integers, for $i = 1, 2, \dots, 1993$;
   (ii) there is no point with both integer coordinates other than $P_i$ and $P_{i+1}$ on the line segment $P_iP_{i+1}$, for $i = 0, 1, \dots, 1992$.

   Prove that for some $i$, $0 \le i \le 1992$, there exists a point $Q$ with coordinates $(q_x, q_y)$ on the line segment joining $P_iP_{i+1}$ such that both $2q_x$ and $2q_y$ are odd integers.



The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com