1-st Asian–Pacific Mathematical Olympiad 1989

1. Let $x_1, x_2, \dots, x_n$ be positive real numbers, and let $S = x_1 + x_2 + \cdots + x_n$. Prove that
$$(1+x_1)(1+x_2)\cdots(1+x_n) \le 1 + S + \frac{S^2}{2!} + \frac{S^3}{3!} + \cdots + \frac{S^n}{n!}.$$

2. Prove that the equation
$$6(6a^2 + 3b^2 + c^2) = 5n^2$$
has no solutions in integers except $a = b = c = n = 0$.

3. Let A_1, A_2, A_3 be three points in the plane, and let $A_4 = A_1$, $A_5 = A_2$. For $n = 1$, 2, and 3, suppose that B_n is the midpoint of A_nA_{n+1}, and suppose that C_n is the midpoint of A_nB_n. Suppose that A_nC_{n+1} and B_nA_{n+2} meet at D_n, and that A_nB_{n+1} and C_nA_{n+2} meet at E_n. Calculate the ratio of the area of triangle $D_1D_2D_3$ to the area of triangle $E_1E_2E_3$.

4. Let S be a set consisting of m pairs (a, b) of positive integers with $1 \le a < b \le n$. Show that there are at least
$$4m \cdot \frac{(m - \frac{n^2}{4})}{3n}$$
triples (a, b, c) such that (a, b), (a, c), and (b, c) belong to S.

5. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ which satisfy the conditions:

   (i) $f(x)$ is strictly increasing;

   (ii) $f(x) + g(x) = 2x$ for all x, where $g(x)$ is the inverse function to $f(x)$.

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