18-th Asian–Pacific Mathematical Olympiad 2006

1. A positive integer n is given. Find the largest nonnegative real number $f(n)$ with the following property: Whenever $a_1, a_2, \ldots, a_n$ are real numbers such that $a_1 + a_2 + \cdots + a_n$ is an integer, there exists an index i for which $|a_i - \frac{1}{2}| \geq f(n)$.

2. Prove that every positive integer can be written as a finite sum of distinct integral powers τ^i ($i \in \mathbb{Z}$) of the golden mean $\tau = \frac{1+\sqrt{5}}{2}$.

3. Let $p \geq 5$ be a prime number and let r be the number of ways of placing p identical checkers on a $p \times p$ checkerboard so that not all checkers are in the same row (but they may be in the same column). Show that r is divisible by p^5.

4. Let P be the midpoint of a chord AB of a given circle O. Circle O_1 is tangent to the line AB at P and tangent to O. A line l through A, different from the line AB, touches O_1 and intersects O again at C. Let Q be the midpoint of the segment BC and O_2 be the circle tangent to BC at Q and tangent to the segment AC. Prove that the circle O_2 is tangent to O.

5. In a circus, there are n clowns who dress and paint themselves up using a selection of 12 distinct colors. Each clown is required to use at least five different colors. One day, the ringmaster of the circus orders that no two clowns have exactly the same set of colors and no more than 20 clowns may use any one particular color. Find the largest number n of clowns so as to make the ringmaster's order possible.

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