17-th Asian–Pacific Mathematical Olympiad 2005

1. Prove that for every irrational number a, there are irrational numbers b and b' such that $a+b$ and ab' are both rational while ab and $a+b'$ are both irrational.

2. Let a,b,c be positive real numbers such that $abc=8$. Prove that
$$\frac{a^2}{\sqrt{(1+a^3)(1+b^3)}}+\frac{b^2}{\sqrt{(1+b^3)(1+c^3)}}+\frac{c^2}{\sqrt{(1+c^3)(1+a^3)}}\geq\frac{4}{3}.$$

3. Show that there exists a triangle which can be cut into 2005 congruent triangles.

4. In a small town, there are $n\times n$ houses indexed by (i,j) for $1\leq i,j\leq n$ with $(1,1)$ being the house at the top left corner, where i and j are the row and column indices, respectively. At time 0, a fire breaks out at the house indexed by $(1,c)$, where $c\leq n/2$. During each subsequent time interval $[t,t+1]$, the fire fighters defend a house which is not yet on fire while the fire spreads to all undefended neighbors of each house which was on fire at time t. (Two houses (i,j) and (k,l) are neighbors if $|i-k|+|j-l|=1$.) Once a house is defended, it remains so all the time. The process ends when the fire can no longer spread. At most how many houses can be saved by fire fighters?

5. In a triangle ABC, points M and N are on sides AB and AC, respectively, such that $MB=BC=CN$. Let R and r denote the circumradius and the inradius of the triangle ABC, respectively. Express the ratio MN/BC in terms of R and r.

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