Bulgarian Mathematical Olympiad 1993, III Round

First Day, 24 april 1993

1. Prove that the equation:
$$x^3 - y^3 = xy + 1993$$
don't have a solution in positive integers.

2. It is given a right-angled triangle ABC. AC and BC are its cathetus. M is the middlepoint of BC. A circle k passing through A and M is tangent to the circumcircle of ABC. N is the second point of intersection of k and the line BC. Prove that the line AN is passing through the middlepoint of the height CH of the triangle ABC.

3. Prove that if a, b, c are positive numbers and $p,q,r \in [0,1]$ and $a+b+c = p+q+r = 1$, then
$$abc \leq \frac{pa+qb+rc}{8}.$$

Second day, 25 april 1993

4. Let a, b, c are positive numbers for which: $9a+11b+29c = 0$. Prove that the equation $4ax^3+bx+c = 0$ have a real root in the closed interval $[0,2]$.

5. It is given an acute-angled triangle ABC for which $BC = AC\sqrt{2}$. Through the vertex C are drawn lines ℓ and m (different from the lines AC and BC), which intersects the line AB respectively at the points L and M in such a way that $AL = MB$. The lines ℓ and m intersects circumcircle of ABC at the points P and Q respectively and the lines PQ and AB intersects each other at N prove that $AB = NB$.

6. It is given a convex hexagon with sidelength equal to 1. Find biggest natural number n for which internally to the hexagon given can be situated n points in such a way that the distance between any two of them is not less than $\sqrt{2}$.

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