Yugoslav IMO Team Selection Test 1969

Belgrade, 1969

1. Given real numbers a_i, b_i ($i = 1, 2, \dots, n$) such that

$$a_1 \geq a_2 \geq \cdots \geq a_n > 0,$$
$$b_1 \geq a_1,$$
$$b_1 b_2 \geq a_1 a_2,$$
$$\dots$$
$$b_1 b_2 \cdots b_n \geq a_1 a_2 \cdots a_n,$$

prove that $b_1 + b_2 + \cdots + b_n \geq a_1 + a_2 + \cdots + a_n$.

2. Let $f(x)$ and $g(x)$ be degree n polynomials, and x_0, x_1, $\dots$, x_n be real numbers such that

$$f(x_0) = g(x_0), f'(x_1) = g'(x_1), f''(x_2) = g''(x_2), \dots, f^{(n)}(x_n) = g^{(n)}(x_n).$$

Prove that $f(x) = g(x)$ for all x.

3. Points A and B move with a constant speed along lines a and b. Two corresponding positions of these points A_1, B_1, and A_2, B_2 are known. FInd the position of A and B for which the length of AB is minimal.

4. Let a and b be two natural numbers such that $a < b$. Prove that in each set of b consecutive positive integers there are two numbers whose product is divisible by ab.

5. Prove that the product of the sines of two opposite dihedrals in a tetrahedron is proportional to the product of the lengths of the edges of these dihedrals.

6. Let E be the set of $n^2 + 1$ closed intervals on the real axis. Prove that there exists a subset of $n + 1$ intervals that are monotonically increasing with respect to inclusion, or a subset of $n + 1$ intervals none of which contains any other interval from the subset.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com