Yugoslav IMO Team Selection Test 1997

Niš, April 13, 1997.

Time allowed 180 *minutes.*
Each problem is worth 25 *points.*

1. Consider a regular n-gon $A_1A_2\dots A_n$ with area S. Let us draw the lines $l_1,l_2,\dots,l_n$ perpendicular to the plane of the n-gon at $A_1,A_2,\dots,A_n$ respectively. Points $B_1,B_2,\dots,B_n$ are selected on lines $l_1,l_2,\dots,l_n$ respectively so that:

   (i) $B_1,B_2,\dots,B_n$ are all on the same side of the plane of the n-gon;

   (ii) Points $B_1,B_2,\dots,B_n$ lie on a single plane;

   (iii) $A_1B_1=h_1$, $A_2B_2=h_2,\dots,A_nB_n=h_n$.

   Express the volume of polyhedron $A_1A_2\dots A_nB_1B_2\dots B_n$ as a function in $S,h_1,\dots,h_n$.

2. Given a natural number k, find the smallest natural number C such that

$$\frac{C}{n+k+1}\binom{2n}{n+k}$$

   is an integer for every integer $n\geq k$.

3. Numbers $1,2,\dots,1997^2$ are written in the cells of a 1997×1997 table. It is allowed to apply the following transformations: exchange places of any two rows or any two columns, or reverse a row or column. (When a row or column is reversed, the first and last entry exchange their positions, so do the second and second last, etc.) Is it possible that, after finitely many such transformations, arbitrary two numbers exchange their positions and no other number changes its position?

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