Yugoslav IMO Team Selection Test 1990

Tuzla, April 22, 1990

1. The sequence (a_n) is given by $a_1 = 4$, $a_2 = 6$ and
$$a_{n+2}a_n = a_{n+1}^2 + 8 \qquad \text{for } n \geq 1.$$
Prove that $9a_n^2 - 128$ is a square of a rational number for all n.

2. Prove the following identity for all $n \in \mathbb{N}$:
$$\left[\sqrt{n} + \sqrt{n+1} + \sqrt{n+2} + \sqrt{n+3}\right] = \left[\sqrt{16n+20}\right].$$

3. Prove that from every set of $n+1$ natural numbers, whose prime factors are in a given set of n prime numbers, one can select several distinct numbers whose product is a perfect square.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com