Yugoslav IMO Team Selection Test 1978

Bečići, 1978

1. Find all integers x, y, z such that $x^2(x^2+y)=y^{z+1}$.

2. Let k_0 be a unit semi-circle with diameter AB. Assume that k_1 is a circle of radius $r_1=1/2$ that is tangent to both k_0 and AB. The circle k_{n+1} of radius r_{n+1} touches k_n, k_0, and AB. Prove that:

   (a) For each $n\in\{2,3,\dots\}$ it holds: $\frac{1}{r_{n+1}}+\frac{1}{r_{n-1}}=\frac{6}{r_n}-4$.

   (b) $\frac{1}{r_n}$ is either a square of an even integer, or twice a square of an odd integer.

3. Let $\mathscr{F}$ be the collection of subsets of a set with n elements such that no element of $\mathscr{F}$ is a subset of another of its elements. Prove that

$$|\mathscr{F}|\leq\binom{n}{[n/2]}.$$

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