Yugoslav IMO Team Selection Test 1977

Velenje, 1977

1. Determine the set of all real numbers α with the following property: For each positive c there exists a rational number $\frac{m}{n}$ ($m \in \mathbb{Z}$, $n \in \mathbb{N}$) different than α such that
$$\left|\alpha - \frac{m}{n}\right| < \frac{c}{n}.$$

2. Determine all 6-tuples (p,q,r,x,y,z) where p, q, r are prime, and x, y, z natural numbers such that $p^{2x} = q^y r^z + 1$.

3. Assume that the equality $2BC = AB + AC$ holds in $\triangle ABC$. Prove that:

   (a) The vertex A, the midpoints M and N of AB and AC respectively, the incenter S, and the circumcenter O belong to a circle k.

   (b) The line TS, where T is the centroid of $\triangle ABC$ is a tangent to k.

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