Yugoslav IMO Team Selection Test 1976

Kragujevac, 1976

1. Prove that for a given convex polygon of area A and perimeter P there exists a circle of radius A/P that is contained in the interior of the polygon.

2. Assume that $2n+1$ positive integers satisfy the following: If we remove any of these integers, the remaining $2n$ integers can be partitioned in two groups of n numbers in each, such that the sum of the numbers in one group is equal to the sum of the numbers in the other. Prove that all of these numbers must be equal.

3. Find the minimal and maximal value of the function

$$f(x,y,z,t) = \frac{ax^2+by^2}{ax+by} + \frac{az^2+bt^2}{az+bt}, \ \ (a>0, b>0),$$

given that $x+z=y+t=1$, and x, y, z, $t \geq 0$.

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