Yugoslav IMO Team Selection Test 1974

Belgrade, 1974

1. Assume that a is a given irrational number.

   (a) Prove that for each positive real number ε there exists at least one integer $q \geq 0$ such that $aq - [aq] < \varepsilon$.

   (b) Prove that for given $\varepsilon > 0$ there exist infinitely many rational numbers p/q such that $q > 0$ and $\left|a - \frac{p}{q}\right| < \frac{\varepsilon}{q}$.

2. Given two directly congruent triangles ABC and $A'B'C'$ in a plane, assume that the circles with centers C and C' and radii CA and $C'A'$ intersect. Denote by $\mathscr{M}$ the transformation that maps $\triangle ABC$ to $\triangle A'B'C'$. Prove that $\mathscr{M}$ can be expressed as a composition of at most three rotations in the following way: The first rotation has the center in some of the vertices of $\triangle ABC$ and maps $\triangle ABC$ to $\triangle A_1B_1C_1$; The second rotation has the center in one of A_1, B_1, C_1, and maps $\triangle A_1B_1C_1$ to $\triangle A_2B_2C_2$; The third rotation has the center in one of A_2, B_2, C_2 and maps $\triangle A_2B_2C_2$ to $\triangle A'B'C'$.

3. Let S be a set of n points P_1, P_2, $\dots$, P_n in a plane such that no three of the points are collinear. Let α be the smallest of the angles $\angle P_iP_jP_k$ ($i \neq j \neq k \neq i$, $i, j, k \in \{1, 2, \dots, n\}$). Find $\max_S \alpha$ and determine those sets S for which this maximal value is attained.

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