Serbia and Montenegro Team Selection Test 2003

Novi Sad, April 20, 2003

Time allowed 3 *hours.*
Each problem is worth 25 *points.*

1. If $p(x)$ is a polynomial, denote by $p^n(x)$ a polynomial
$$\underbrace{p(p(\cdots p}_{n}(x)\cdots)).$$
Prove that the polynomial $p^{2003}(x)-2p^{2002}(x)+p^{2001}(x)$ is divisible by $p(x)-x$.

2. Each edge and each diagonal of the convex n-gon (where $n \geq 3$) is colored in red or blue. Prove that the vertices of the n-gon can be labeled as $A_1, A_2, \dots, A_n$ in such a way that one of the following two conditions is satisfied:
   - 1° all segments $A_1A_2, A_2A_3, \dots, A_{n-1}A_n, A_nA_1$ are of the same color;
   - 2° there exists a number k, $1 < k < n$ such that the segments A_1A_2, A_2A_3, $\dots, A_{k-1}A_k$ are blue, and the segments $A_kA_{k+1}, \dots, A_{n-1}A_n, A_nA_1$ are red.

3. Let M and N be the distinct points in the plane of the triangle ABC such that $AM : BM : CM = AN : BN : CN$. Prove that the line MN contains the circumcenter of $\triangle ABC$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com