26-th Vietnamese Mathematical Olympiad 1988

First Day

1. There are 1988 birds in 994 cages, two in each cage. Every day we change the arrangement of the birds so that no cage contains the same two birds as ever before. What is the greatest possible number of days we can keep doing so?

2. Suppose $P(x) = a_n x^n + \cdots + a_1 x + a_0$ be a real polynomial of degree $n > 2$ with $a_n = 1$, $a_{n-1} = -n$, $a_{n-2} = \frac{n^2 - n}{2}$ such that all the roots of P are real. Determine the coefficients a_i.

3. The plane is partitioned into congruent equilateral triangles such that any two of them which are not disjoint have either a common vertex or a common side. Is there a circle containing exactly 1988 points in its interior?

Second Day

4. A bounded sequence $(x_n)_{n\geq 1}$ of real numbers satisfies
$$x_n + x_{n+1} \geq 2x_{n+2} \qquad \text{for all } n \geq 1.$$
Prove that this sequence has a finite limit.

5. Suppose that ABC is an acute triangle such that $\tan A$, $\tan B$, $\tan C$ are the three roots os the equation $x^3 - px^2 + qx - p = 0$, where $q \neq 1$. Show that $p \leq 3\sqrt{3}$ and $q > 1$.

6. Let a, b, c be three pairwise skew lines in space. Prove that they have a common perpendicular if and only if $\mathscr{S}_a \circ \mathscr{S}_b \circ \mathscr{S}_c$ is a reflection in a line, where $\mathscr{S}_x$ denotes the reflection in line x.

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