24-th Vietnamese Mathematical Olympiad 1986

First Day

1. Let $1/2 \leq a_1, a_2, \ldots, a_n \leq 5$ be given real numbers and let $x_1, x_2, \ldots, x_n$ be real numbers satisfying $4x_i^2 - 4a_i x_i + (a_i - 1)^2 \leq 0$. Prove that
$$\sqrt{\sum_{i=1}^{n} \frac{x_i^2}{n}} \leq \sum_{i=1}^{n} \frac{x_i}{n} + 1.$$

2. Let R, r be respectively the circumradius and inradius of a regular $1986-$gonal pyramid. Prove that
$$\frac{R}{r} \geq 1 + \frac{1}{\cos(\pi/1986)}$$
and find the total area of the surface of the pyramid when the equality occurs.

3. Suppose $M(y)$ is a polynomial of degree n such that $M(y) = 2^y$ for $y = 1, 2, \ldots, n+1$. Compute $M(n+2)$.

Second Day

4. Let $ABCD$ be a square of side a. An equilateral triangle AMB is constructed in the plane through AB perpendicular to the plane of the square. A point S moves on AB. Let P be the projection of M on SC and E, O be the midpoints of AB and CM respectively.
   (a) Find the locus of P as S moves on AB.
   (b) Find the maximum and minimum lengths of SO.

5. Find all $n > 1$ such that the inequality
$$\sum_{i=1}^{n} x_i^2 \geq x_n \sum_{i=1}^{n-1} x_i$$
holds for all real numbers $x_1, x_2, \ldots, x_n$.

6. A sequence of positive integers is constructed as follows: the first term is 1, the following two terms are $2, 4$, the following three terms are $5, 7, 9$, the following four terms are $10, 12, 14, 16$, etc. Find the n-th term of the sequence.

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