20-th Vietnamese Mathematical Olympiad 1982

First Day

1. Determine a quadric polynomial with intergral coefficients whose roots are $\cos 72^\circ$ and $\cos 144^\circ$.

2. For a given parameter m, solve the equation
$$x(x+1)(x+2)(x+3)+1-m=0.$$

3. Let be given a triangle ABC. Equilateral triangles BCA_1 and BCA_2 are drawn so that A and A_1 are on one side of BC, whereas A_2 is on the other side. Points B_1, B_2, C_1, C_2 are analogously defined. Prove that
$$S_{ABC} + S_{A_1B_1C_1} = S_{A_2B_2C_2}.$$

Second Day

4. Find all positive integers x, y, z such that $2^x + 2^y + 2^z = 2336$.

5. Let p be a positive integer and q, z be real numbers with $0 \le q \le 1$ and $q^{p+1} \le z \le 1$. Prove that
$$\prod_{k=1}^{p} \left| \frac{z-q^k}{z+q^k} \right| \le \prod_{k=1}^{p} \left| \frac{1-q^k}{1+q^k} \right|.$$

6. Let $ABCDA'B'C'D'$ be a cube (where $ABCD$ and $A'B'C'D'$ are faces and AA', BB', CC', DD' are edges). Consider the four lines AA', BC, $D'C'$ and the line joining the midpoints of BB' and DD'. Show that there is no line which cuts all the four lines.

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