40-th Vietnamese Mathematical Olympiad 2002

First Day - March 12

1. Solve the equation $\sqrt{4-3\sqrt{10-3x}} = x-2$.

2. An isosceles triangle ABC with $AB = AC$ is given on the plane. A variable circle (O) with center O on the line BC passes through A and does not touch either of the lines AB and AC. Let M and N be the second points of intersection of (O) with lines AB and AC, respectively. Find the locus of the orthocenter of triangle AMN.

3. Let be given two positive integers m,n with $m < 2001$, $n < 2002$. Let distinct real numbers be written in the cells of a 2001×2002 board (with 2001 rows and 2002 columns). A cell of the board is called *bad* if the corresponding number is smaller than at least m numbers in the same column and at least n numbers in the same row. Let s denote the total number of bad cells. Find the least possible value of s.

Second Day - March 13

4. Let a,b,c be real numbers for which the polynomial x^3+ax^2+bx+c has three real zeroes. Prove that

$$12ab+27c \leq 6a^3+10(a^2-2b)^{3/2}.$$

When does equality occur?

5. Determine all positive integers n for which the equation

$$x+y+u+v = n\sqrt{xyuv}$$

has a solution in positive integers x,y,u,v.

6. For a positive integer n, consider the equation

$$\frac{1}{x-1}+\frac{1}{4x-1}+\frac{1}{9x-1}+\cdots+\frac{1}{n^2x-1} = \frac{1}{2}.$$

   (a) Prove that, for every n, this equation has a unique root greater than 1, which is denoted by x_n.

   (b) Prove that the limit of x_n is 4 as n approaches infinity.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com