Turkish IMO Team Selection Test 2006

First Day – April 1, 2006

1. Find the largest area of a heptagon inscribed in a unit circle and having two perpendicular diagonals.

2. In how many different ways can a $2\times n$ rectangle be partitioned into rectangles with integer side lengths?

3. If x,y,z are positive numbers with $xy+yz+zx=1$, show that

$$\frac{27}{4}(x+y)(y+z)(z+x) \geq \left(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}\right)^2 \geq 6\sqrt{3}.$$

Second Day – April 2, 2006

4. A sequence (x_n) of integers satisfies $x_{n+1}=x_1^2+x_2^2+\cdots+x_n^2$ for $n\geq 1$. Find the smallest value of x_1 for which 2006 divides x_{2006}.

5. A point $Q\neq A,B$ is given on the circle with diameter AB. Point H is the orthogonal projection of Q on AB. The circle with center Q and radius QH intersects the circle with diameter AB at C and D. Show that CD bisects QH.

6. In a university entrance examination with 2006000 students, each student makes a list of 12 colleges from the 2006 colleges in total. It turns out that, for any six students, there exist two colleges such that each of the six students included at least one of these two colleges into his/her list. An *extensive* list is a list which includes at least one college from each student's list.

   (a) Show that there always exists an extensive list of 12 colleges.

   (b) Show that the students can choose their lists so that no extensive list of fewer than 12 colleges can be found.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com