# Turkish IMO Team Selection Test 2005

*First Day – April 2, 2005*

1. Find all functions $f : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ satisfying the conditions

$$4f(x) \geq 3x \quad \text{and} \quad f(4f(x) - 3x) = x \quad \text{for all } x \geq 0.$$

2. Let $N$ be the midpoint of the side $AB$ of a triangle $ABC$ with $\angle A > \angle B$. Let $D$ be a point on the ray $AC$ such that $CD = BC$ and $P$ be a point on the ray $DN$ which lies on the same side of $BC$ as $A$ and satisfies the condition $\angle PBC = \angle A$. The lines $PC$ and $AB$ intersect at $E$, and the lines $BC$ and $DP$ intersect at $T$. Determine the value of $\dfrac{BC}{TC} - \dfrac{EA}{EB}$.

3. Initially the numbers 1 through 2005 are marked. A finite set of marked consecutive integers is called a *block* if it is not contained in any larger set of marked consecutive integers. In each step we select a set of marked integers which does not contain the first or last element of any block, unmark the selected integers, and mark the same number of consecutive integers starting with the integer two greater than the largest marked integer. What is the minimum number of steps necessary to obtain 2005 single integer blocks?

*Second Day – April 3, 2005*

4. Show that for any integer $n \geq 2$ and all integers $a_1, a_2, \ldots, a_n$, the product $\prod_{i<j}(a_j - a_i)$ is divisible by $\prod_{i<j}(j - i)$.

5. Let $ABC$ be a triangle such that $\angle A = 90^\circ$ and $\angle B < \angle C$. The tangent at $A$ to its circumcircle $\Gamma$ meets the line $BC$ at $D$. Let $E$ be the reflection of $A$ across $BC$, $X$ the foot of the perpendicular from $A$ to $BE$, and $Y$ the midpoint of $AX$. Let the line $BY$ meet $\Gamma$ again at $Z$. Prove that the line $BD$ is tangent to the circumcircle of triangle $ADZ$.

6. We are given 5040 balls in $k$ different colors, where the number of balls of each color is the same. The balls are put into 2520 bags so that each bag contains two balls of different colors. Find the smallest $k$ such that, however the balls are distributed into the bags, we can arrange the bags around a circle so that no two balls of the same color are in two neighboring bags.



www.imomath.com

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com