Swiss IMO Team Selection Test 1997

May 17

1. A finite sequence of integers $a_0, a_1, \ldots, a_n$ is called *quadratic* if $|a_k - a_{k-1}| = k^2$ for $1 \leq k \leq n$.

   (a) Prove that for any two integers b and c, there exist a natural number n and a quadratic sequence with $a_0 = b$ and $a_n = c$.

   (b) Find the smallest natural number n for which there exists a quadratic sequence with $a_0 = 0$ and $a_n = 1997$.

2. Let $ABCD$ be a convex quadrilateral. Find the necessary and sufficient condition for the existence of point P inside the quadrilateral such that the triangles ABP, BCP, CDP, DAP have the same area.

3. A 6×6 square has been tiled by 18 dominoes. Show that there exists a line that divides the square into two parts, each of which is also tiled by dominoes.

4. Let v and w be two randomly chosen roots of the equation $z^{1997} - 1 = 0$ (all roots are equiprobable). Find the probability that $\sqrt{2+\sqrt{3}} \leq |v + w|$.

www.imomath.com

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com