

Swedish Mathematical Competition 1995

Final Round

November 18, 1995

1. All pages of a magazine are numbered and printed on both sides. One sheet with two sides is missing. The numbers of the remaining pages sum to 963. How many pages did the magazine have originally and which pages are missing?
2. Botvid left home between 4 and 5 for a short visit to Amanda. When he came back between 5 and 6, he found that the hands of the clock had changed places. What time was it?
3. Let a, b, x, y be positive numbers with $a + b + x + y < 2$. Given that

$$a + b^2 = x + y^2 \quad \text{and} \quad a^2 + b = x^2 + y,$$

show that $a = x$ and $b = y$.

4. The product of three positive numbers is 1 and their sum is greater than the sum of their inverses. Prove that one of these numbers is greater than 1, while the other two are smaller than 1.
5. On a circle with center O and radius r are given points A, B, C, D in this order such that AB, BC and CD have the same length s and the length of AD is $s + r$. Assume that $s < r$. Determine the angles of quadrilateral $ABCD$.
6. Signals used for communication are binary sequences of length 10. Unfortunately, the receiving device got broken so that it cannot distinguish between two signals unless those differ in more than five places. What is the largest possible number of signals that can still be used to prevent ambiguities?