45-th Swedish Mathematical Competition 2005

Final Round
Lund, November 19, 2005

1. Find all integer solutions x, y of the equation $(x + y^2)(x^2 + y) = (x + y)^3$.

2. There are 12 people in a line in a bank. When the desk closes, the people form a new line at a newly opened desk. In how many ways can they do this in such a way that none of the 12 people changes his/her position in the line by more than one?

3. In a triangle ABC the bisectors of angles A and C meet the opposite sides at D and E respectively. Show that if the angle at B is greater than $60°$, then $AE + CD < AC$.

4. The zeroes of a fourth degree polynomial $f(x)$ form an arithmetic progression. Prove that the three zeroes of the polynomial $f'(x)$ also form an arithmetic progression.

5. Every cell of a 2005×2005 square board is colored white or black so that every 2×2 subsquare contains an odd number of black cells. Show that among the corner cells there is an even number of black ones. How many ways are there to color the board in this manner?

6. A regular tetrahedron of edge length 1 is orthogonally projected onto a plane. Find the largest possible area of its image.

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