# 44-th Swedish Mathematical Competition 2004

## Final Round
Linköping, November 20, 2004

1. Two circles in the plane, both of radius $R$, intersect at a right angle. Compute the area of the intersection of the interiors of the two circles.

2. In one country there are coins of value $1, 2, 3, 4$ or $5$. Nisse wants to buy a pair of shoes. While paying, he tells the seller that he has 100 coins in the bag, but that he does not know the exact number of coins of each value. "Fine, then you will have the exact amount", the seller responds. What is the price of the shoes, and how did the seller conclude that Nisse would have the exact amount?

3. A dunction $f$ satisfies $f(x) + xf(1-x) = x^2$ for all real $x$. Determine $f$.

4. If $0 < v < \dfrac{\pi}{2}$ and $\tan v = 2v$, decide whether $\sin v < \dfrac{20}{21}$.

5. A square of side $n \geq 2$ is divided into $n^2$ unit squares ($n \in \mathbb{N}$). One draws $n-1$ lines so that the interior of each of the unit squares is cut by at least one of these lines.

    (a) Give an example of such a configuration for some $n$.

    (b) Show that some two of the lines must meet inside the square.

6. Prove that every convex $n$-gon contains a quadrilateral of area at least $\frac{1}{2}$.