43-rd Swedish Mathematical Competition 2003

Final Round

Göteborg, November 22, 2003

1. If x, y, z, w are nonnegative real numbers satisfying

$$y = x - 2003, \quad z = 2y - 2003, \quad w = 3z - 2003,$$

find the smallest possible value of x and the values of y, z, w corresponding to it.

2. In a lecture hall some chairs are placed in rows and columns, forming a rectangle. In each row there are 6 boys sitting and in each column there are 8 girls sitting, whereas 15 places are not taken. What can be said about the number of rows and that of columns?

3. Find all real solutions x of the equation $[x^2 - 2x] + 2[x] = [x]^2$.

4. Determine all polynomials P with real coefficients such that

$$1 + P(x) = \frac{1}{2}\left(P(x-1) + P(x+1)\right) \quad \text{for all real } x.$$

5. Given two positive numbers a, b, how many non-congruent plane quadrilaterals are there such that $AB = a$, $BC = CD = DA = b$ and $\angle B = 90°$?

6. Consider an infinite square board with an integer written in each square. Assume that for each square the integer in it is equal to the sum of its neighbor to the left and its neighbor above. Assume also that there exists a row R_0 in the board such that all numbers in R_0 are positive. Denote by R_1 the row below R_0, by R_2 the row below R_1 etc. Show that for each $N \geq 1$ the row R_N cannot contain more than N zeroes.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imo.org.yu